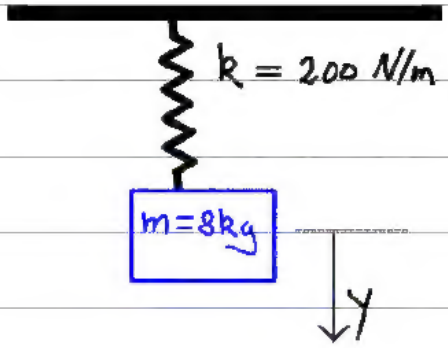
Meriam 8/3 : Forced Vibrations (undamped)

Consider a simple vibrating system:

$k/m = 200/8 = 25 = \omega_n^2$

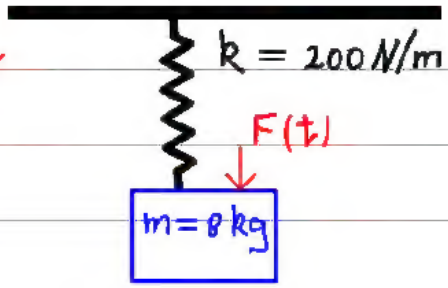
$\omega_n = 5 \ s^{-1}$



$y(t) = A \sin(5t + \phi)$

→ vibration only occurs at ω_n

- Suppose we now start to push m ?
- We consider only a certain "pushing profile": sinusoid
- Two possible mechanisms



(a) Indirect contact $\boxed{d(t) = d_0 \sin(\omega_0 t)}$

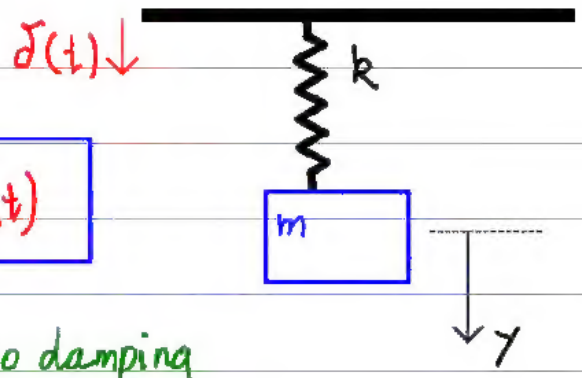
or

(b) Direct contact: $\boxed{F(t) = F_0 \sin(\omega_0 t)}$

Note: ω_0 = forcing frequency
ω_n = natural frequency

We physically expect large oscillations if $\omega_0 \approx \omega_n$

ω_{crit} = "critical frequency"



(a) Indirect contact $\boxed{\delta(t) = \delta_0 \sin(\omega_0 t)}$

Assume no damping

Solution procedure:

1/ Initially, ignore the external excitation:

- select origin as neutral point
- define +ve direction
- displace mass a small +ve distance y.

2/ Add in the external excitation:

$$\Sigma F = -ky + k\delta_0 \sin(\omega_0 t) = m\ddot{y}$$

3/ Put equation in standard form:

$$m\ddot{y} + ky = k\delta_0 \sin(\omega_0 t)$$

$$\ddot{y} + \omega_n^2 y = \frac{k\delta_0}{m} \sin(\omega_0 t) \quad , \quad \omega_n^2 = k/m$$

Solution:

$$\boxed{y(t) = \left(\frac{1}{1 - (\omega_0/\omega_n)^2}\right)\delta_0 \sin\omega_0 t} + C_1 \sin(\omega_n t) + C_2 \cos(\omega_n t)$$

"steady-state" solution

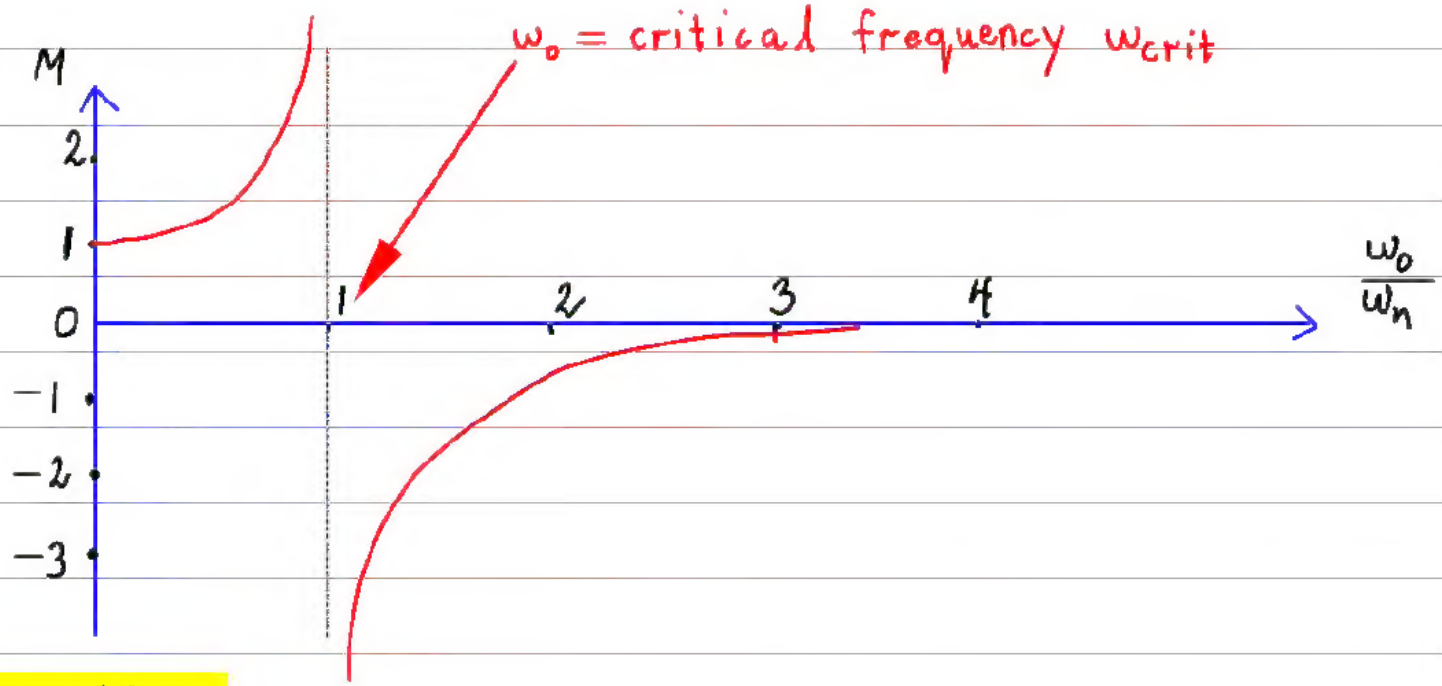
complementary solution

steady-state solution:

$$y(t) = \underbrace{\left(\frac{1}{1 - (\omega_0/\omega_n)^2}\right)}_{\text{Magnification factor "M"}} \underbrace{\delta_0 \sin\omega_0 t}_{\text{static deflection}}$$

where "static deflection" is oscillation amplitude at $\omega_0 \to 0$

Note:
- At very small values of ω_0, $M = 1$, $\sum \vec{F} = m\vec{a} \approx 0$
- If ω_0 is close to $\omega_n \to$ large oscillation amplitudes
- If $\omega_0 > \omega_n$, M is $-ve$ (?)
- If $\omega_0 \gg \omega_n$, $M \to 0$



More Notes:
- You can add a phase factor to earthquake: $\delta_0 \sin(\omega_0 t + \phi)$
- We can have a rotational forced vibration:

$$\sum M_0 = I_0 \ddot{\theta}$$

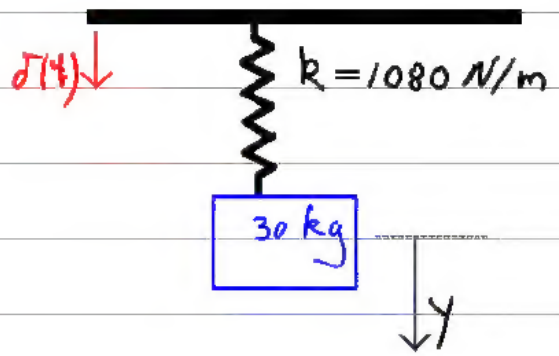
$$\theta(t) = \theta_{static} \times M \times \sin(\omega_0 t)$$

δ_0/R

$\delta_0 \sin(\omega_0 t) \leftrightarrow k$

E.g.

Given: $\delta(t) = \underbrace{2.3\,cm}_{\delta_0} \sin(\omega_0 t)$



Find range of ω_0 to give vibration amplitude ≤ 7.5 cm.

Solution:

- select origin at neutral position
- $\sum F_y = -1080y + k\delta_0 \sin\omega_0 t = m\ddot{y}$

$$\ddot{y} + \underbrace{\frac{k}{m}}_{\omega_n^2} y = \frac{k}{m}\delta_0 \sin(\omega_0 t)$$

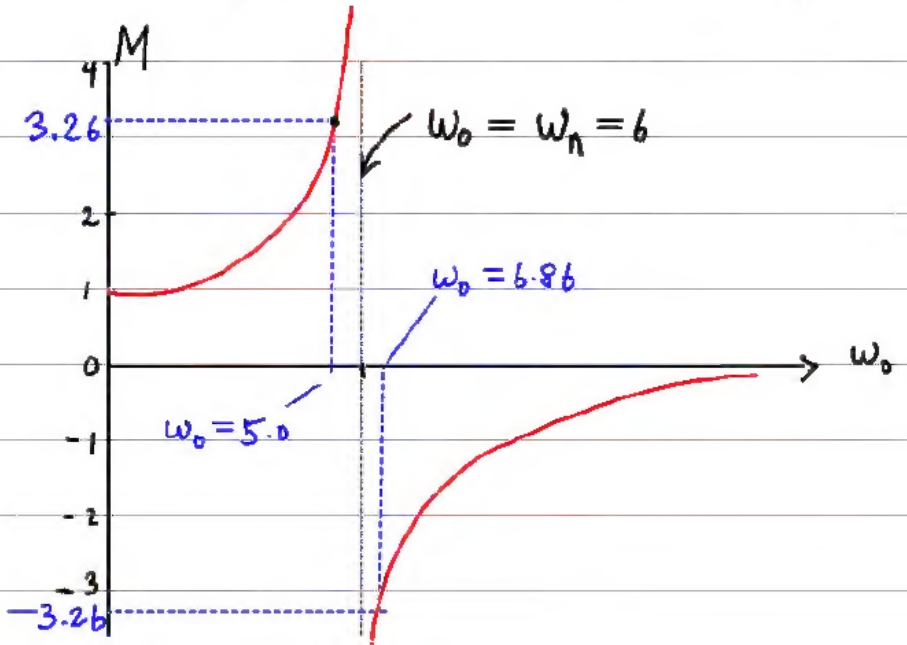
$\rightarrow \omega_n = \sqrt{1080/30} = 6\ s^{-1}$

steady-state solution: $y(t)$ = static deflection $\times$ M $\times \sin\omega_0 t$

$$y(t) = \underbrace{\delta_0}_{0.023\,m} \times \frac{1}{1 - \omega_0^2/6^2} \times \sin\omega_0 t$$

amplitude ≤ 0.075 m $\rightarrow$ $(0.023)(|M|) \leq 0.075$ $\quad |M| \leq 3.26$

$$|M| = \left|\frac{1}{1 - \omega_0^2/6^2}\right| = 3.26 \rightarrow \omega_0 = 5.0 \text{ or } 6.86\ s^{-1}$$



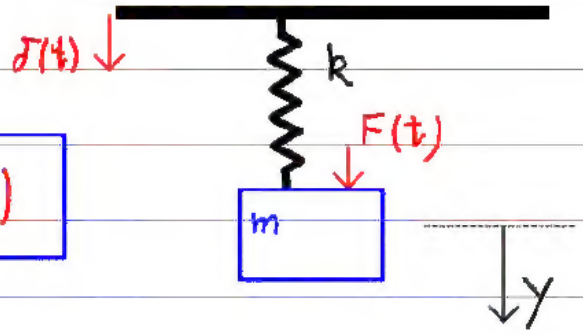
we need

$\omega_0 < 5$ or $\omega_0 > 6.86$

$\rightarrow |M| < 3.26$

$\rightarrow$ amp < 0.075 m

Two Possible Mechanisms for Forced Vibrations



(a) Indirect contact $\boxed{\delta(t) = \delta_0 \sin(\omega_0 t)}$

or

(b) Direct contact: $\boxed{F(t) = F_0 \sin(\omega_0 t)}$

Solution to (b) Origin at neutral point, displace mass & release

$$\Sigma F_y = -ky + k\delta(t) = m\ddot{y} \qquad (k\delta(t) \to F(t))$$

$$\ddot{y} + \frac{k}{m} y = \frac{k\delta(t)}{m} \qquad (k\delta(t) \to F(t))$$

$k\delta_0 \sin\omega_0 t \sim F_0 \sin\omega_0 t \rightarrow k\delta_0 \sim F_0$

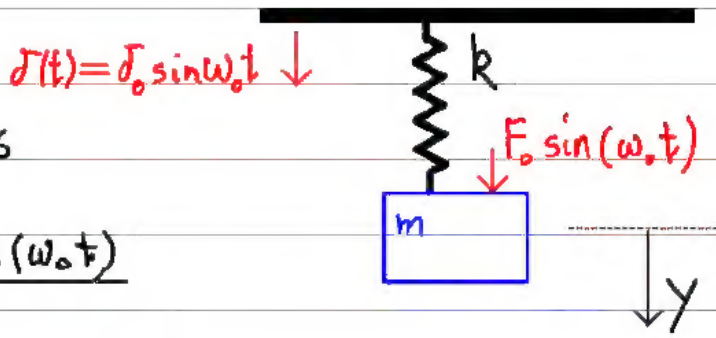
$$\boxed{y(t) = \left(\frac{1}{1 - (\omega_0/\omega_n)^2}\right) \times \delta_0 \left(= \frac{F_0}{k}\right) \times \sin\omega_0 t}$$

M — static deflection

→ To get $y(t)$ or $\theta(t)$, you need:

- $\omega_n \rightarrow M$
- static deflection based on amplitude of $F(t)$ or $\delta(t)$

Summary

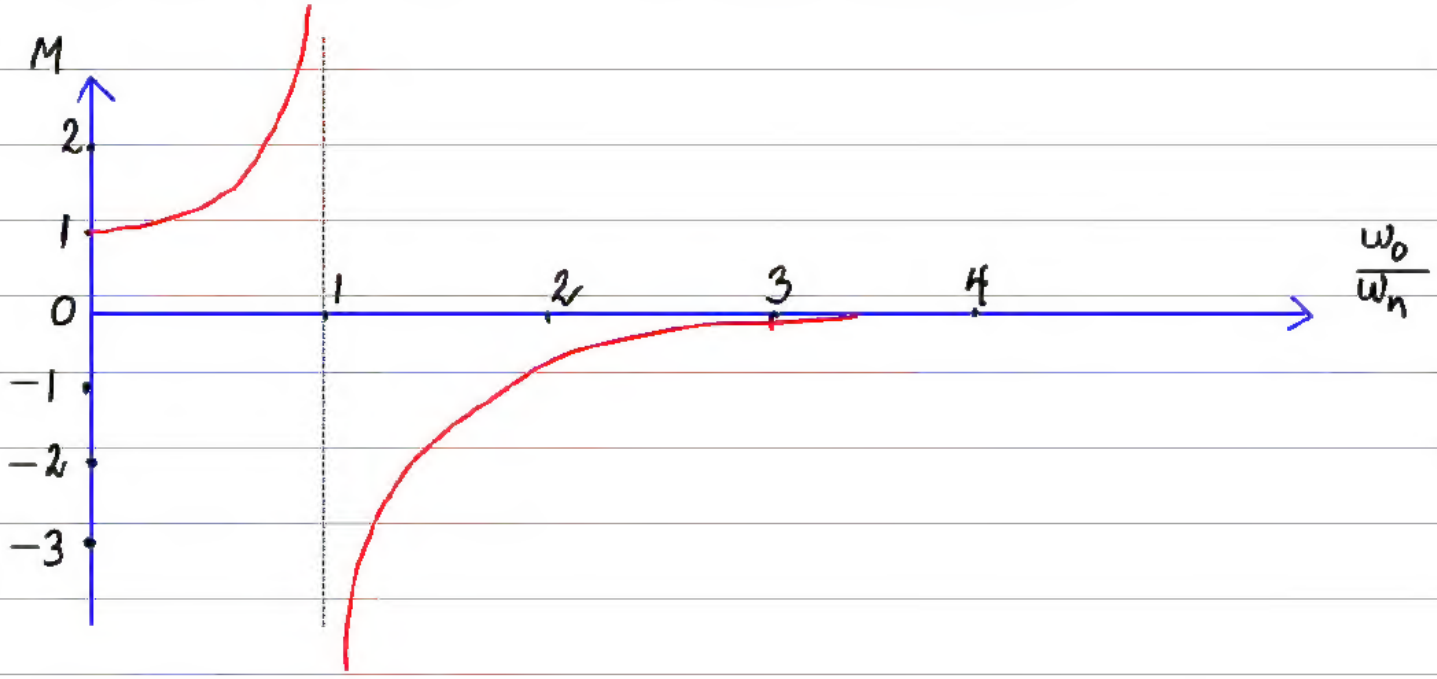


- Forced undamped vibrations

- $\ddot{y} + \frac{k}{m} y = \frac{F_0 \leftarrow (k\,\delta_0) \sin(\omega_0 t)}{m}$

- $y(t) = \underbrace{\left(\frac{1}{1 - (\omega_0/\omega_n)^2}\right)}_{M} \underbrace{\delta_0}_{F_0/k} \sin \omega_0 t$

M — static deflection



Next Lecture

- Vibration sample problems: Meriam 8/3, 8/4